II Semester M.Sc. Examination, June 2016
(CBCS)
MATHEMATICS
M203T : Topology – II

Time : 3 Hours Max. Marks : 70

Instructions : *i) Answer **any five full** questions.*
*ii) **All** questions carry **equal** marks.*

1. a) Show that a closed subset of a compact space is compact.
   b) Define :
      i) A countable compact space
      ii) Sequentially compact space.

      Prove that every sequentially compact space is countably compact. Is the converse true ? Explain.
   c) If $f : X \to Y$ is a continuous mapping of a locally compact space $(X, \mathcal{T})$ onto a topological space $(Y, \mathcal{T}^*)$, then prove that $(Y, \mathcal{T}^*)$ is locally compact.

   (3+7+4)

2. a) Prove that every second axiom space is a first axiom space and hence show that converse is false.
   b) Prove that Lindeloff property is topological.
   c) Prove that a metric space (X, d) is countably compact iff every countable open cover has a finite subcover.

   (6+3+5)

3. a) Define the projections on the product space $X \times Y$ and show that they are continuous and open.
   b) Show that $X \times Y$ is second countable iff X and Y are second countable.
   c) Prove that if A is closed in $(X, \mathcal{T})$ and B is closed in $(Y, \mathcal{T})$ then $A \times B$ is closed in the product topology and conversely.

   (4+4+6)

4. a) Define :
      i) T_0-space
      ii) T_1-space.

      Give an example of a T_0-space which is not a T_1-space.

P.T.O.

b) Prove that in a T_0-space the closure of distinct points are distinct and conversely.

c) Show that a point 'x' in a T_1-space (X, τ) is a limit point of a subset A of X if and only if every open set containing x contains infinitely many distinct points of A.

(4+4+6)

5. a) Define a T_3-space. Show that a metric space is T_3-space.

b) Prove that every T_3-space is a T_2-space. Is the converse true ? Justify.

c) Define a Tychnoff space. Show that a Tychnoff space is a regular space.

(5+4+5)

6. a) Define a normal space. Show that a T_2-space need not be normal.

b) Show that a regular Lindeloff space is normal.

(4+10)

7. a) State and prove the Urysohn's lemma.

b) Show that a normal space is regular if and only if it is completely regular.

(10+4)

8. a) Define complete normal space. Prove that a space is completely normal iff every subspace is normal.

b) Prove that every metric space is normal.

c) Define para compact space. Prove that every para compact space is normal.

(6+4+4)